

Potential–density pairs for spherical galaxies and bulges: the influence of scalar fields.

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ABSTRACT

A family of potential–density pairs has been found for spherical halos and bulges of galaxies in the Newtonian limit of scalar–tensor theories of gravity. The scalar field is described by a Klein–Gordon equation with a source that is coupled to the standard Poisson equation of Newtonian gravity. The net gravitational force is given by two contributions: the standard Newtonian potential plus a term stemming from massive scalar fields. General solutions have been found for spherical systems. In particular, we compute potential–density pairs of spherical galactic systems, and some other astrophysical quantities that are relevant to generating initial conditions for spherical galaxy simulations.

Key words: stellar dynamics – elementary particles – Galaxy: kinematics and dynamics – Galaxy: halo – Galaxy: bulge – Galaxy: structure

1 INTRODUCTION

In recent years there has been important progress in understanding the dynamics that led to the formation of galaxies. Two and three dimensional N -body simulations of galaxies and protogalaxies have been computed using millions of particles, giving a more realistic view of how galaxies, quasars, and black holes could have formed (Barnes 1998; Barnes & Hernquist 1992).

The composition of Universe at the time galaxies formed could theoretically be very varied, including visible and dark baryonic matter, non–baryonic dark matter, neutrinos, and many cosmological relics stemming from symmetry breaking processes predicted by high energy physics (Kolb & Turner 1990). All these particles and fields, if present, should have played a role in structure formation. Accordingly, recent independent observational data; CMBR at various angular scales (de Bernardis et al 2000; Bennett et al 2003), type Ia supernovae (Riess et al 1998; Perlmutter et al 1999; Riess et al 2001), as well as the 2dF Galaxy Redshift survey (Peacock 2002; Efsthathiou 2002), suggest that $\Omega = \Omega_\Lambda + \Omega_m \approx 1$, or $\Omega_\Lambda \approx 0.7$ and $\Omega_m \approx 0.3$, implying the existence of dark energy and dark matter, respectively. One particular candidate for dark energy is a scalar field usually called quintessence (Caldwell, Dave & Steinhard 1998). In this way galaxies are expected to possess dark components and, in accordance with the rotation curves of stars and gas around the centres of spirals, they might be in the form of

halos (Ostriker & Peebles 1973) and must contribute to at least 3 to 10 times the mass of the visible matter (Kolb & Turner 1990).

In order to construct numerical galaxies in a consistent way using gravity and particle physics components, one must find the correct physical scheme. The theoretical framework to explain the existence of dark components finds its origin in theories of elementary particles physics, with the addition of the action of gravity. There exist many theories (grand unification schemes, string theories, brane worlds, etc) that involve such physics, but scalar–tensor theories (STT) of gravity are typically found to represent classical effective descriptions of such original theories (Green, Schwarz & Witten 1988). In this way, the scalar fields of these theories are the natural candidates to be the quintessence field (Caldwell, Dave & Steinhard 1998; Boisseau, Esposito–Farese, Polarski & Starobinsky 2000; Amendola 2001), as a remnant of some cosmological function that contributes $\Omega_\Lambda \approx 0.7$ today. It has been even suggested that the quintessence field is the scalar field that also acts on local planetary scales (Fujii 2000) or on galactic scales (Matos & Guzman 2001). Moreover, massive scalar fields might account to the dark matter components of galaxies in the form of halos.

Motivated by the above arguments, we study in the present report some STT effects in galactic systems. Specifically, we compute potential–density pairs coming from such theories in their Newtonian approximation. The results pre-

sented here will be useful in constructing numerical galaxies that are consistent with grand unification schemes.

This paper is organised as follows: in section 2 we present the Newtonian approximation of a general scalar-tensor theory of gravity. Solutions are presented in terms of integrals of Green functions, and some expressions for the velocities and dispersions of stars in galaxies are given. In section 3, solutions for a family of potential-density pairs (Dehnen 1993) and the Navarro, Frenk, and White (NFW) model (Navarro et al 1996-97) are presented, and some relevant observational quantities are computed. In section 4 we present conclusions.

2 SCALAR FIELDS AND THE NEWTONIAN APPROXIMATION

Let us consider a typical scalar-tensor theory given by the following Lagrangian

$$\mathcal{L} = \frac{\sqrt{-g}}{16\pi} \left[-\phi R + \frac{\omega(\phi)}{\phi} (\partial\phi)^2 - V(\phi) \right] + L_M(g_{\mu\nu}), \quad (1)$$

from which we get the gravitational equations,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{\phi} \left[8\pi T_{\mu\nu} + \frac{1}{2}Vg_{\mu\nu} + \frac{\omega}{\phi} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \frac{\omega}{\phi} (\partial\phi)^2 g_{\mu\nu} + \phi_{;\mu\nu} - g_{\mu\nu} \square\phi \right], \quad (2)$$

and the scalar field equation

$$\square\phi + \frac{\phi V' - 2V}{3 + 2\omega} = \frac{1}{3 + 2\omega} [8\pi T - \omega'(\partial\phi)^2], \quad (3)$$

where $(\cdot)' \equiv \frac{\partial}{\partial\phi}$. In the past we have studied (Cervantes-Cota & Dehnen 1995), among others, this type of non-minimal coupling between gravity and matter fields in a cosmological context in the very early Universe, where all relativistic effects have to be taken into account. In the present study, however, we want to consider the influence of scalar fields within galaxies, and therefore we need to describe the theory in its Newtonian approximation, that is, where gravity and the scalar fields are weak and velocities of stars are non-relativistic. We expect to have small deviations of the scalar field around the background field, defined here as $\langle\phi\rangle = G_N^{-1} = 1$. If one defines the perturbation $\bar{\phi} \equiv \phi - 1$, the Newtonian approximation gives (Helbig 1991)

$$R_{00} = \frac{1}{2}\nabla^2 h_{00} = 4\pi\rho - \frac{1}{2}\nabla^2 \bar{\phi}, \quad (4)$$

$$\nabla^2 \bar{\phi} - m^2 \bar{\phi} = -8\pi\alpha\rho, \quad (5)$$

where we have introduced

$$\frac{\phi V' - 2V}{3 + 2\omega} = m^2 \bar{\phi} - m^2 k \bar{\phi}^2 + \dots,$$

and $\alpha \equiv 1/(3 + 2\omega)$. In the above expansion we have set the cosmological constant term equal to zero, since on galactic scales its influence should be negligible. We only consider the influence of dark matter due to the boson field of mass m governed by Eq. (5), that is a Klein-Gordon equation with a source.

Note that Eq. (4) can be cast as a Poisson equation for $\psi \equiv (1/2)(h_{00} + \bar{\phi})$,

$$\nabla^2 \psi = 4\pi\rho. \quad (6)$$

The new Newtonian potential is now given by

$$\Phi_N \equiv \frac{1}{2}h_{00} = \psi - \frac{1}{2}\bar{\phi}. \quad (7)$$

The next step is to find solutions for this new Newtonian potential given a density profile, that is, to find the so-called potential-density pairs. First we present the solutions for point-like masses and second for general density distributions.

2.1 Point-like masses

The solution of these equations is known and given by

$$\begin{aligned} \bar{\phi} &= 2\alpha u_\lambda, \\ h_{00} &= -2u - 2\alpha u_\lambda, \end{aligned} \quad (8)$$

where

$$\begin{aligned} u &= \sum_s \frac{m_s}{|\mathbf{r} - \mathbf{r}_s|}, \\ u_\lambda &= \sum_s \frac{m_s}{|\mathbf{r} - \mathbf{r}_s|} e^{-|\mathbf{r} - \mathbf{r}_s|/\lambda}, \end{aligned} \quad (9)$$

with m_s a source mass, and the total gravitational force on a particle of mass m_i is

$$\sum \mathbf{F} = -\frac{1}{2}\nabla h_{00} = m_i \mathbf{a}, \quad (10)$$

where $\lambda = \hbar/mc$ is the Compton wavelength of the effective mass (m) of some elementary particle (boson) given through the potential $V(\phi)$ and $\omega(\phi)$. In what follows we will use λ instead of m^{-1} . This mass can have a range of values depending on particular particle physics models. The potential u is the Newtonian part and u_λ is the dark matter contribution which is of the Yukawa type. There are two limits: On the one hand, for if $r \gg \lambda$ (or $\lambda \rightarrow 0$) one recovers the Newtonian theory of gravity. On the other hand, for if $r \ll \lambda$ (or $\lambda \rightarrow \infty$) one again obtains the Newtonian theory, but now with a rescaled Newtonian constant, $G \rightarrow G_N(1 + \alpha)$. There are stringent constraints on the possible λ - α values determined by measurements on local scales (Fischbach & Talmadge 1999).

In the past the above solutions have been used to solve the missing mass problem in spirals (Sanders 1984; Eckhardt 1993) as an alternative to considering a distribution of dark matter. This was done assuming that most of the galactic mass is located in the galactic centre, and then considering the centre to be a point source. In our present investigation we do not avoid dark matter, since our model predicts that bosonic dark matter produces, through a scalar field associated to it, a modification of Newtonian gravity theory. This dark matter is presumably clumped in the form of dark halos. Therefore we will consider in what follows that a dark halo is spherically distributed along an observable spiral and beyond, having some density profile (section 3). Next, we compute the potentials, and some astrophysical quantities, for general halo density distributions.

2.2 General density distributions

General solutions to Eqs. (5) and (6) can be found in terms of the corresponding Green functions

$$\psi = - \int d\mathbf{r}_s \frac{\rho(\mathbf{r}_s)}{|\mathbf{r} - \mathbf{r}_s|} + \text{B.C.}, \quad (11)$$

$$\bar{\phi} = 2\alpha \int d\mathbf{r}_s \frac{\rho(\mathbf{r}_s) e^{-|\mathbf{r} - \mathbf{r}_s|/\lambda}}{|\mathbf{r} - \mathbf{r}_s|} + \text{B.C.}, \quad (12)$$

and the new Newtonian potential is

$$\begin{aligned} \Phi_N &= \psi - \frac{1}{2}\bar{\phi} \\ &= - \int d\mathbf{r}_s \frac{\rho(\mathbf{r}_s)}{|\mathbf{r} - \mathbf{r}_s|} \\ &\quad - \alpha \int d\mathbf{r}_s \frac{\rho(\mathbf{r}_s) e^{-|\mathbf{r} - \mathbf{r}_s|/\lambda}}{|\mathbf{r} - \mathbf{r}_s|} + \text{B.C.} \end{aligned} \quad (13)$$

The first term of Eq. (13), given by ψ , is the contribution of the usual Newtonian gravitation (without scalar fields), while information about the scalar field is contained in the second term, that is, arising from the influence function determined by the Klein–Gordon Green function, where the coupling ω (α) enters as part of a source factor.

Given that we are interested in spherical galaxies and bulges in what follows we will only consider the case of spherical symmetry. Additionally, we use flatness boundary conditions (B.C.) at infinity, such that the boundary terms in Eqs. (11)–(12) are zero. Moreover, regularity conditions must be applied to spatial points where the potentials are singular. For spherical systems these conditions mean that $d\psi/dr = d\bar{\phi}/dr = 0$ at the origin. Accordingly, performing the integrals in these equations, we obtain ψ for a system of radius R ,

$$\psi(r_a) = -\frac{M}{a} \begin{cases} \frac{m(r_a)}{r_a} + o(r_a) & ; r_a < R_a \\ \frac{m(R_a)}{r_a} & ; r_a \geq R_a \end{cases}, \quad (14)$$

and for $\bar{\phi}$

$$\bar{\phi}(r_a) = \frac{2M}{a} \eta \times \begin{cases} \frac{e^{-r_a/\lambda_a}}{r_a} p(r_a) + \frac{\sinh(r_a/\lambda_a)}{r_a} q(r_a) & ; r_a < R_a \\ \frac{e^{-r_a/\lambda_a}}{r_a} p(R_a) & ; r_a \geq R_a, \end{cases} \quad (15)$$

where we have defined the functions

$$\begin{aligned} m(r_a) &\equiv \int_0^{r_a} dx x^2 \tilde{\rho}(x), \\ o(r_a) &\equiv \int_{r_a}^{R_a} dx x \tilde{\rho}(x), \\ p(r_a) &\equiv \int_0^{r_a} dx x^2 \frac{\sinh(x/\lambda_a)}{x/\lambda_a} \tilde{\rho}(x), \\ q(r_a) &\equiv \int_{r_a}^{R_a} dx x^2 \frac{\exp(-x/\lambda_a)}{x/\lambda_a} \tilde{\rho}(x), \end{aligned} \quad (16)$$

and we have written the density as

$$\rho(r) = \frac{M}{4\pi a^3} \tilde{\rho}(r_a). \quad (17)$$

Here M is a reference or scaling mass and a is a scaling length that we use to define the following quantities: $r_a \equiv r/a$, $R_a \equiv R/a$, $\lambda_a \equiv \lambda/a$, and $\eta \equiv \alpha/\lambda_a$. Then, the new Newtonian potential can be written as

$$\Phi_N(r_a) = -\frac{M}{a} F(r_a), \quad (18)$$

where

$$F(r_a) = \begin{cases} \frac{m(r_a)}{r_a} + o(r_a) + \eta \left[\frac{e^{-r_a/\lambda_a}}{r_a/\lambda_a} p(r_a) + \frac{\sinh(r_a/\lambda_a)}{r_a/\lambda_a} q(r_a) \right] & ; r_a < R_a \\ \frac{m(R_a)}{r_a} + \eta \frac{e^{-r_a/\lambda_a}}{r_a/\lambda_a} p(R_a) & ; r_a \geq R_a. \end{cases} \quad (19)$$

2.3 Intrinsic and projected properties

We now want to present the intrinsic and observable quantities of interest that characterize the steady state of spherical galaxies and bulges. These include the circular velocity, velocity dispersion, distribution function, projected velocity dispersion, and projected density.

The circular velocity for a spherical galactic system is, from the above equations,

$$v_c^2 = r \frac{d\Phi_N}{dr} = -\frac{M}{a} r_a \frac{dF(r_a)}{dr_a}. \quad (20)$$

For $r_a < R_a$ one obtains straightforwardly the circular velocities for stars inside a dark halo of size R . For $r_a \geq R_a$ one gets

$$\begin{aligned} \frac{a}{M} v_c^2 &= \frac{m(R_a)}{r_a} + \alpha [o(R_a) \sinh(R_a/\lambda_a) - Q(R_a)] \\ &\quad \times \frac{e^{-r_a/\lambda_a}}{r_a/\lambda_a} \left(1 + \frac{r_a}{\lambda_a}\right), \end{aligned} \quad (21)$$

where

$$Q(r_a) \equiv \frac{1}{\lambda_a} \int_0^{r_a} dx x o(x) \cosh(x/\lambda_a), \quad (22)$$

which gives the circular velocities outside the halo (if there are stars there). From Eqs. (19) and (20) one obtains, after redefinition of the multiplicative constant, the expression found in the past for rotational velocities due to point masses (Sanders 1984).

A full description of stars in a galaxy is given by the collisionless Boltzmann equation, which after being integrated over all velocities yields the Jeans equations relating the potential and density of a system to its velocity dispersion (Binney & Tremaine 1994). In particular, the velocity dispersion profile is important in determining the stationary state of the system. For equilibrium stationary states we found the velocity dispersion for a spherically symmetric stellar system, using Eq. (20), to be

$$\bar{v}_r^2 = -\frac{M}{a} \frac{1}{\tilde{\rho}(r_a)} \int_{r_a}^{\infty} dx \tilde{\rho}(x) \frac{dF(x)}{dx}, \quad (23)$$

where we have assumed that $\bar{v}_r^2 = \bar{v}_\theta^2$ and $\bar{v}_r = \bar{v}_\theta = 0$. We have used similar expressions to generate initial conditions for proto–galaxies in order to study the influence of scalar fields on the transfer of angular momentum during collisions (Rodríguez–Meza et al 2001).

The distribution function of the isotropic models can be calculated using the Abel transform (Binney & Tremaine 1994). First, we define $\mathcal{E} = -(a/M)E$, E the total energy, and $\Psi(r_a) = -(a/M)\Phi_N(r_a)$, and we

obtain r as a function of Ψ by inverting. The mass density is

$$\rho(r) = 4\pi \left(\frac{M}{a}\right)^{3/2} \int_0^\Psi f(\mathcal{E}) \sqrt{2(\Psi - \mathcal{E})} d\mathcal{E}. \quad (24)$$

The inversion of this equation is done using the Eddington formula and we obtain the distribution function

$$f(\mathcal{E}) = \frac{1}{\sqrt{8\pi^2}} \left(\frac{a}{M}\right)^{3/2} \left[\int_0^\mathcal{E} \frac{d^2\rho}{d\Psi^2} \frac{d\Psi}{\sqrt{\mathcal{E} - \Psi}} + \frac{1}{\sqrt{\mathcal{E}}} \left(\frac{d\rho}{d\Psi}\right)_{\Psi=0} \right]. \quad (25)$$

The projected velocity dispersion for the isotropic system is given by (Binney & Tremaine 1994)

$$\sigma_p^2(R) = \frac{2}{\Sigma(R)} \int_R^\infty \rho(r) \bar{v}_r^2 \frac{r dr}{\sqrt{r^2 - R^2}}, \quad (26)$$

where $\Sigma(R) = 2 \int_R^\infty dr \rho(r) r / \sqrt{r^2 - R^2}$ is the projected density. This also gives us the cumulative surface density

$$S(R) = 2\pi \int_0^R \Sigma(R) R dR. \quad (27)$$

3 POTENTIAL-DENSITY PAIRS

In the past some potential-density pairs have been studied within the framework of Newtonian gravity. The Jaffe (Jaffe 1983) and Hernquist density models (Hernquist 1990) have been generalised by Dehnen (1993) by introducing a free parameter (γ) that determines particular density models for galaxy description. Another density profile of interest, obtained from N-body cosmological simulations, has been proposed by NFW (Navarro et al 1996-97). In this section we will consider both models in order to study the influence of scalar fields on spherical systems.

3.1 Dehnen's profile

We use the family of density profiles for spherical halos and bulges of galaxies proposed by Dehnen (1993),

$$\rho(r) = \frac{(3-\gamma)M}{4\pi} \frac{a}{r^\gamma(r+a)^{4-\gamma}}, \quad (28)$$

where a is a scaling radius and M denotes the total mass. The Hernquist profile corresponds to $\gamma = 1$ and Jaffe's to $\gamma = 2$.

Solving the Poisson equation without a scalar field, *i.e.* for Newtonian gravity, the potential that corresponds to the density of Eq. (28) is (Dehnen 1993)

$$\Phi_N = \psi = \frac{M}{a} \times \begin{cases} -\frac{1}{2-\gamma} \left[1 - \left(\frac{r}{r+a}\right)^{2-\gamma} \right] & ; \gamma \neq 2 \\ \ln \frac{r}{r+a} & ; \gamma = 2. \end{cases} \quad (29)$$

Due to the influence of the scalar field, however, $\psi(r)$ does not represent the total gravitational potential of the system. The effective Newtonian potential is now $\Phi_N = \frac{1}{2} h_{00}(r)$ determined by Eq. (13). Thus, using Eq. (28) for the density, we obtain for $r_a < R_a$

$$\begin{aligned} \frac{a}{M} \Phi_N &= - \int_{r_a}^{R_a} dx \frac{m(x)}{x^2} \\ &\quad - \eta \left[\frac{e^{-r_a/\lambda_a}}{r_a/\lambda_a} p(r_a) + \frac{\sinh(r_a/\lambda_a)}{r_a/\lambda_a} q(r_a) \right], \end{aligned} \quad (30)$$

and for $r_a \geq R_a$

$$\frac{a}{M} \Phi_N = - \frac{m(R_a)}{r_a} - \eta \frac{e^{-r_a/\lambda_a}}{r_a/\lambda_a} p(R_a). \quad (31)$$

Using Eq. (28) one finds explicitly the functions m , p , and q :

$$m(r_a) = (3-\gamma) \int_0^{r_a} dx \frac{x^{2-\gamma}}{(x+1)^{4-\gamma}} = \left(\frac{r_a}{r_a+1}\right)^{3-\gamma}, \quad (32)$$

$$p(r_a) = (3-\gamma)\lambda_a \int_0^{r_a} dx \frac{x^{1-\gamma}}{(x+1)^{4-\gamma}} \sinh(x/\lambda_a) \quad (33)$$

$$\begin{aligned} &= (3-\gamma) \left[\frac{1}{3-\gamma} r_a^{3-\gamma} {}_2F_1(3-\gamma, 4-\gamma; 4-\gamma, -r_a) \right. \\ &\quad \left. + \frac{1}{6\lambda_a^2} \frac{1}{5-\gamma} r_a^{5-\gamma} {}_2F_1(5-\gamma, 4-\gamma; 6-\gamma, -r_a) + \dots \right], \end{aligned}$$

$$q(r_a) = (3-\gamma)\lambda_a \int_{r_a}^{R_a} dx \frac{x^{1-\gamma}}{(x+1)^{4-\gamma}} e^{-x/\lambda_a} \quad (34)$$

$$\begin{aligned} &= (3-\gamma) \left\{ \left[\frac{\lambda_a}{2-\gamma} R_a^{2-\gamma} {}_2F_1(2-\gamma, 4-\gamma; 3-\gamma, -R_a) \right. \right. \\ &\quad - \frac{1}{3-\gamma} R_a^{3-\gamma} {}_2F_1(3-\gamma, 4-\gamma; 4-\gamma, -R_a) \\ &\quad + \frac{1}{2\lambda_a} \frac{1}{4-\gamma} R_a^{4-\gamma} {}_2F_1(4-\gamma, 4-\gamma; 5-\gamma, -R_a) - \dots \Big] \\ &\quad - \left[\frac{\lambda_a}{2-\gamma} r_a^{2-\gamma} {}_2F_1(2-\gamma, 4-\gamma; 3-\gamma, -r_a) \right. \\ &\quad - \frac{1}{3-\gamma} r_a^{3-\gamma} {}_2F_1(3-\gamma, 4-\gamma; 4-\gamma, -r_a) \\ &\quad \left. \left. + \frac{1}{2\lambda_a} \frac{1}{4-\gamma} r_a^{4-\gamma} {}_2F_1(4-\gamma, 4-\gamma; 5-\gamma, -r_a) - \dots \right] \right\}, \end{aligned}$$

where ${}_2F_1(a, b; c; z)$ is a hypergeometric function.

For $r_a < R_a$ the integral in the first term of Eq. (30) can be evaluated, giving for the standard Newtonian potential

$$\psi = -\frac{M}{a} \times \begin{cases} \frac{1}{2-\gamma} \left[\left(\frac{R_a}{R_a+1}\right)^{2-\gamma} - \left(\frac{r_a}{r_a+1}\right)^{2-\gamma} \right] & ; \gamma \neq 2 \\ \ln \frac{R_a}{R_a+1} - \ln \frac{r_a}{r_a+1} & ; \gamma = 2, \end{cases} \quad (35)$$

which for $R_a \rightarrow \infty$ reduces to Eq. (29).

The circular velocity is, for $r_a < R_a$,

$$\begin{aligned} \frac{a}{M} v_c^2 &= \frac{m(r_a)}{r_a} + \eta \left[\left(1 + \frac{r_a}{\lambda_a}\right) \frac{e^{-r_a/\lambda_a}}{r_a/\lambda_a} p(r_a) \right. \\ &\quad \left. + \left(1 - \frac{r_a}{\lambda_a} \coth(r_a/\lambda_a)\right) \frac{\sinh(r_a/\lambda_a)}{r_a/\lambda_a} q(r_a) \right], \end{aligned} \quad (36)$$

and for $r_a \geq R_a$

$$\frac{a}{M} v_c^2 = \frac{m(R_a)}{r_a} + \eta \left(1 + \frac{r_a}{\lambda_a}\right) \frac{e^{-r_a/\lambda_a}}{r_a/\lambda_a} p(R_a). \quad (37)$$

The first right-hand-side term in both Eqs. (36) and (37) is the contribution from ψ and their second term is the contribution from the scalar field. The latter term depends on R_a , λ_a and $\eta = \alpha/\lambda_a$, rather than on α alone, which is typical for forces exerted on point systems (Fischbach & Talmadge 1999). Therefore, the stringent constraints on α found for point masses at local scales are less severe here, since the above terms can still yield a large amplitude, even when α is small. One sees that for $r \gg R$, λ ,

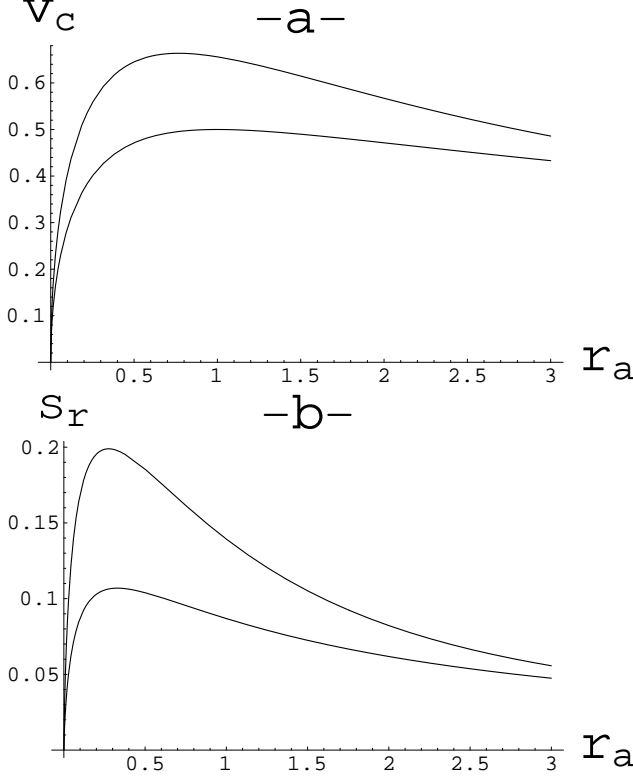


Figure 1. Rotation curves (a) and velocity dispersion curves (b) for the Hernquist model. The upper curves were computed with $\lambda_a = 1$ and $\eta = 1$, while the lower curves were obtained for $\lambda_a = 1$ and $\eta = 0$, i.e., without a scalar field.

the influence of scalar field decays exponentially, recovering the standard Newtonian result.

The velocity dispersion can be computed by substituting Eqs. (36) and (37) into Eq. (23), and this determines completely the initial conditions for spherical galaxies. Fig. 1 shows the circular velocity and the velocity dispersion curves ($s_r = \frac{\sigma}{M} v_r^2$) for the Hernquist model ($\gamma = 1$) for $\eta = 1$ (upper curves) and $\eta = 0$ (lower curves), i.e., the upper curves are computed with the scalar field taken into account and the lower curves without a scalar field. In the limit $\lambda \rightarrow \infty$ one recovers the Newtonian result with a modified G as expected. The computations were done with $R \rightarrow \infty$.

3.2 NFW profile

N -body cosmological simulations (Navarro et al 1996-97) suggest a universal density profile for spherical dark halos of the following type:

$$\rho(r) = \frac{\rho_{\text{crit}} \delta_c}{(r/r_s)(1 + r/r_s)^2}, \quad (38)$$

where r_s is a scale radius, δ_c is a dimensionless constant, and ρ_{crit} is the critical density (for closure) of the Universe. This profile is the same as Dehnen's with $\gamma = 1$ (Hernquist model) for $r \ll r_s$, but it differs at other radii.

This halo density profile was obtained for cosmological simulations in which the Newtonian theory of gravity was used. Because we are considering $\lambda \ll H^{-1}$, then all predictions of such cosmological simulations must be taken into

account; this is because in the present formalism for $r \gg \lambda$ one gets the Newtonian theory.

Solving Eqs. (5) and (6) with the density profile (38), we get for $r_a < R_a$,

$$\begin{aligned} \frac{a}{M} \Phi_N(r_a) = & -\frac{m(r_a)}{r_a} - o(r_a) \\ & -\eta \left[\frac{e^{-r_a/\lambda_a}}{r_a/\lambda_a} p(r_a) + \frac{\sinh(r_a/\lambda_a)}{r_a/\lambda_a} q(r_a) \right] \end{aligned} \quad (39)$$

and for $r_a \geq R_a$

$$\frac{a}{M} \Phi_N(r_a) = -\frac{m(R_a)}{r_a} - \eta \frac{e^{-R_a/\lambda_a}}{r_a/\lambda_a} p(R_a), \quad (40)$$

where the reference length $a = r_s$. The circular velocity is given by Eqs. (36) and (37) where the functions $m(r_a)$, $o(r_a)$, $p(r_a)$, and $q(r_a)$ are given by

$$m(r_a) = \frac{1}{1+r_a} + \ln(1+r_a) - 1, \quad (41)$$

$$o(r_a) = \frac{1}{1+r_a} - \frac{1}{1+R_a}, \quad (42)$$

$$\begin{aligned} p(r_a) = & -\frac{\lambda_a}{1+r_a} \sinh(r_a/\lambda_a) \\ & + \cosh(1/\lambda_a) \left[\text{Chi}\left(\frac{1+r_a}{\lambda_a}\right) - \text{Chi}\left(\frac{1}{\lambda_a}\right) \right] \\ & - \sinh(1/\lambda_a) \left[\text{Shi}\left(\frac{1+r_a}{\lambda_a}\right) - \text{Shi}\left(\frac{1}{\lambda_a}\right) \right], \end{aligned} \quad (43)$$

$$\begin{aligned} q(r_a) = & \lambda_a \frac{e^{-r_a/\lambda_a}}{1+r_a} - \lambda_a \frac{e^{-R_a/\lambda_a}}{1+R_a} \\ & + e^{1/\lambda_a} \left[\text{Ei}\left(-\frac{1+r_a}{\lambda_a}\right) - \text{Ei}\left(-\frac{1+R_a}{\lambda_a}\right) \right], \end{aligned} \quad (44)$$

where $\text{Ei}(z)$, $\text{Shi}(z)$, and $\text{Chi}(z)$ are the exponential, hyperbolic sine and hyperbolic cosine integrals, respectively.

In Fig. 2 we plot the circular velocity and its dispersion for the NFW profile. The values of λ_a and η are the same as in Fig. 1. The upper curves correspond to the case in which we take into account the scalar field, while the lower curves were computed for standard Newtonian gravity. We took the size of the distribution of mass to be $R = 20 r_s$.

The influence of the scalar field is to enhance, especially in the inner regions, both the velocity curves and dispersion. These effects are a little more pronounced in the Hernquist model than in the NFW model.

4 DISCUSSION AND CONCLUSIONS

We have found potential–density pairs of spherical galactic systems within the context of linearised scalar–tensor theories of gravitation. The influence of massive scalar fields is given by $\tilde{\phi}$, determined by Eq. (15). General expressions have been given for circular velocities and dispersions of stars in the spherical system; by stars we mean probe particles that follow the dark halo potential. Specifically, these results were used to find potential–density pairs for the generic Dehnen density parametrisation, as well as for the NFW profile. In general the contribution due to massive scalar fields

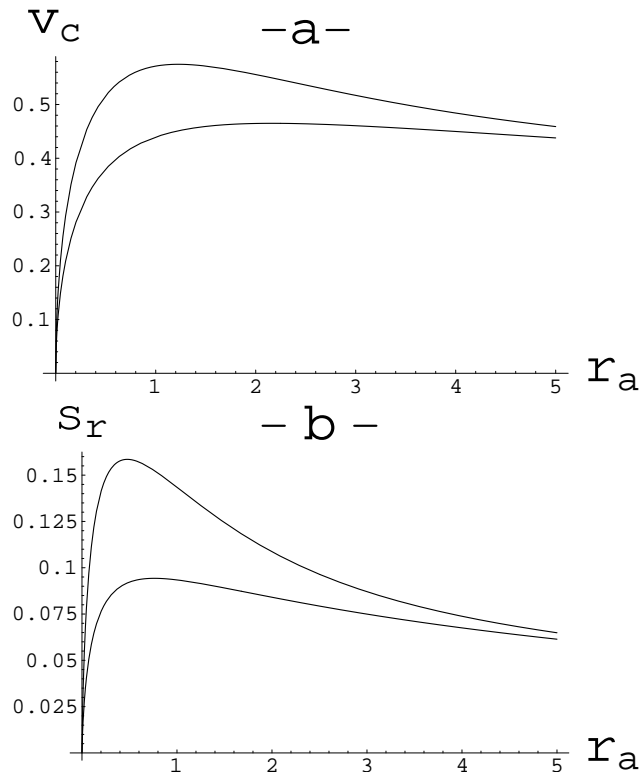


Figure 2. Rotation curves (a) and velocity dispersion curves (b) for the NFW density profile. $\lambda_a = 1$ and upper (lower) curve was computed with $\eta = 1$ ($\eta = 0$).

is non-trivial, see for instance Eqs. (36-37), and interestingly, forces on circular orbits of stars depend on the amplitude terms R_a , λ_a and $\eta = \alpha/\lambda_a$. This means that even when local experiments force α to be a very small number (Fischbach & Talmadge 1999), the amplitude of forces exerted on stars is not necessarily very small and may contribute significantly to the dynamics of stars. Alternatively, one may interpret the local Newtonian constant as given by $(1 + \alpha)\langle\phi\rangle^{-1}$, instead of being given by $\langle\phi\rangle^{-1}$ (1 in our convention). In this case, the local measurement constraints are automatically satisfied, and at scales larger than λ one sees a reduction of $1/(1 + \alpha)$ in the Newtonian constant. If this were the case, then the upper curves in figures 1 and 2 would have to be multiplied by $1/(1 + \alpha)$.

In the past, different authors have used point solutions, Eqs. (8) and (9), to solve the missing mass problem encountered in the rotational velocities of spirals and in galaxy cluster dynamics (Sanders 1984; Eckhardt 1993). These models were used as an alternative to avoid dark matter. Indeed, for single galaxies one can adjust the parameters (α , λ) to solve these problems without the need for dark matter. However, these models do not provide a good description of the systematics of galaxy rotational curves because they predict the scale λ to be independent of the galactic luminosity, and this conflicts with observations for different galactic sizes (Aguirre et al 2001) unless one assumes various λ 's, and hence various fundamental masses, m , one for each galaxy size. Such a particle spectrum is not expected from theoretical arguments; it represents a considerable fine

tuning of masses. This criticism would also apply to our models. The purpose of the present investigation was, however, not to present a model alternative to dark matter in order to solve the missing mass problem, but to compute the influence of scalar-field dark matter distributed in the form of a dark halo. This contribution, together with bulges and disks, will give rise to a flat velocity curve, as in Newtonian mechanics. Yet, the influence of the scalar field from STT yields some profile modification, especially in inner regions, as is shown in figs. 1 and 2.

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